

Star tracks in the ghost condensate

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Abstract

We consider the infrared modification of gravity by ghost condensate. Naively, in this scenario one expects sizeable modification of gravity at distances of order 1000 km, provided that the characteristic time scale of the theory is of the order of the Hubble time. However, we argue that this is not the case. The main physical reason for the conspiracy is a simple fact that the Earth (and any other object in the Universe) has velocity of at least of order $10^{-3}c$ with respect to the rest frame of ghost condensate. Combined with strong retardation effects present in the ghost sector, this fact implies that no observable modification of the gravitational field of nearby objects occurs. Instead, the physical manifestation of ghost condensate is the presence of “star tracks” — narrow regions of space with growing gravitational and ghost fields inside — along the trajectory of any massive object. We briefly discuss the possibilities to observe these tracks.

1 Introduction

Emerging evidence for the accelerated expansion of the Universe triggered interest in the non-standard theories of gravity in which gravitational interactions get modified in the infrared. To some extent the motivation is that in these theories, unlike in the case of pure cosmological constant, physics responsible for the cosmic acceleration may manifest itself in observations at smaller distance scales (e.g., in Lunar Ranging experiments [1, 2]).

Naively, the most straightforward way to modify gravity at distance scale r_c would be to give a graviton a mass $m_g \sim r_c^{-1}$. However, conventional massive gravity suffers either from the presence of ghosts or from the loss of predictivity because of strong coupling at unacceptably low energy scales [3]. Similar problems arise in multi-dimensional models, and it is not clear whether there exists a consistent *quantum* brane world theory where gravity is modified in the infrared and predictive power is not lost at unacceptably low energy scale.

Recently, an example of a theory which does not suffer from the above problems has been constructed [4]. This theory, dubbed “ghost condensate”, is somewhat similar to the Fierz–Pauli massive gravity. The difference is that the Fierz–Pauli mass term breaks reparametrization invariance completely, while in the ghost condensate theory only the time reparametrization invariance

$$t \rightarrow t + \xi(t, x)$$

is broken, while the invariance under (possibly time-dependent) spatial diffeomorphisms is kept intact.

As a result, the latter theory becomes formally reparametrization invariant after a single Stückelberg field is introduced, as opposed to four Stückelberg fields in the Fierz–Pauli gravity. The key difference between these two theories is that in the case of ghost condensate, decoupling limit exists in which gravity is switched off while the Stückelberg sector is still described by a well defined low-energy effective theory valid up to a certain energy scale M .

The price one pays is that the Lorentz invariance is not preserved by ghost condensate; for instance, at the quadratic level one effectively adds to the Einstein theory the “mass term” of the form

$$\int dt d^3x \frac{1}{8} M^4 h_{00}^2 . \quad (1)$$

As a consequence of this violation of the Lorentz invariance, the dispersion law for the Stückelberg field¹ π has rather peculiar form,

$$\omega^2 = \frac{\alpha}{M^2} k^4 ,$$

where α is a dimensionless parameter of the theory. Another manifestation of the fact that the Lorentz invariance is broken is that modification of gravity in the infrared is not characterized by a single length scale r_c . Instead, there is a length scale r_c , equal to

$$r_c = \frac{\sqrt{2} M_{Pl}}{M^2} \quad (2)$$

and a time scale t_c given by

$$t_c = \frac{2 M_{Pl}^2}{\alpha M^3} . \quad (3)$$

As discussed in Ref. [4], for a massive source *at rest* the length scale r_c determines the characteristic distance at which the gravitational potential starts to deviate from the Newtonian one, while t_c determines the characteristic time needed for this deviation to show up.

¹In what follows, we somewhat loosely refer to this field as a ghost. It is worth stressing however, that this is not a ghost field in the usual sense, i.e. the sign in front of its kinetic term in the action is positive.

Naively, this implies that, assuming that $\alpha \sim 1$ and t_c is of the order of the present age of the Universe, $t_U \sim 15$ Gyr, one might expect a sizeable modification of gravity due to ghost condensate at length scales of order 1000 km.

However, it was noted already in Ref. [4] that the retardation effects are very strong in the ghost sector. The point is that it is the whole history of a system that determines its actual gravitational potential in the presence of ghost condensate. The purpose of this paper is to better understand this feature and thus reconsider possible observational signatures of ghost condensate. It is worth noting that for the moment, consequences of ghost condensate with $t_c \sim t_U$ for present day cosmology have not been elaborated yet (see, however, Ref. [5] where ghost condensate was used to construct a model of inflation with quite unusual perturbation spectra). Still, we believe that the question we address is of relevance, since ghost condensate is an interesting infrared modification of gravity whose consistency is beyond any doubt.

Surprisingly, we find that the above naive expectation is incorrect and it is not excluded that we live in the Universe with $t_c \sim t_U$ and $r_c \sim 1000$ km and have not noticed that so far. To understand how that could be, one recalls a simple fact that objects in the Universe are not at rest. Instead, solar system and other stars in our Galaxy rotate around the center of the Galaxy with typical velocity of order 10^{-3} , while the Galaxy itself moves in the local cluster of galaxies with the velocity of the same order of magnitude. A well-known observational consequence of this motion is the dipole anisotropy of the cosmic microwave background.

This implies that all stellar objects have velocity of at least of the same order of magnitude with respect to the rest frame of ghost condensate². Now, it takes time of order t_c for the modification of the gravitational potential to occur. Consequently, the effect of ghost condensate which we can observe on the Earth now is not a modification, say, of the gravitational field of the Sun, but the “ghost” tail of the potential of a star which was located nearby (in the rest frame of ghost condensate) time t_c ago. The Universe with ghost condensate can be thought of as a kind of a bubble chamber where all moving massive objects leave long (and, as we will see later, narrow) tracks in which ghost field and gravitational potential are perturbed. The time delay between the moment when the object passes a given point in

²A priori one may think that this velocity can be significantly larger if the rest frame of the CMB has a finite velocity with respect to the rest frame of the ghost condensate. On the other hand we expect that one of the effects of the Hubble friction is to slow down this overall motion of the CMB. Throughout this paper we assume that the rest frame of the CMB coincides with the rest frame of the ghost condensate, leaving the study of whether it may be not the case for future.

space and the appearance of the track around this point is of order t_c .

This observation is our main result. The rest of this paper is organized as follows. In section 2 we derive a general expression for the gravitational potential of a massive source in the presence of ghost condensate (in the Newtonian approximation). In section 3 we consider specific examples, namely a potential of a moving point-like source and the effect of the finite size of the source. Section 4 contains preliminary discussion of phenomenological implications of our calculations. Technical details can be found in Appendix.

2 Gravitational potential in the presence of ghost condensate

Throughout this paper we consider small metric perturbations near flat Minkowski space-time

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$$

where $\eta_{\mu\nu}$ is the Minkowski metric with the signature $(+, -, -, -)$. Excitations of ghost condensate are described by a real scalar field π , so, in the linear regime, the modification of gravity takes place only in the scalar sector. In the conformal Newtonian gauge the scalar part of the metric perturbation $h_{\mu\nu}$ has the following non-zero components

$$h_{00} = 2\Phi, \quad h_{ij} = 2\Psi\delta_{ij}.$$

The quadratic Lagrangian describing the coupled system of scalar metric and ghost condensate excitations is

$$\mathcal{L} = \mathcal{L}_{EH} + \mathcal{L}_{gh} + \mathcal{L}_s,$$

where \mathcal{L}_{EH} is the quadratic Einstein action in the Newtonian gauge, \mathcal{L}_{gh} is the quadratic action for ghost condensate and \mathcal{L}_s is the source term. Switching to the momentum space and taking the Newtonian limit $\omega^2 \ll k^2$, one has $\Phi = \Psi$ and the resulting Lagrangian in the rest frame of ghost condensate takes the following form [4]

$$\mathcal{L} = \frac{1}{2} (\pi_c \ \Phi_c) \begin{pmatrix} \omega^2 - \alpha^2 k^4 / M^2 & -im\omega \\ im\omega & -k^2 + m^2 \end{pmatrix} \begin{pmatrix} \pi_c \\ \Phi_c \end{pmatrix} + \mathcal{L}_s \quad (4)$$

where the canonically normalized gravitational potential Φ_c is related to the conventional one in the following way

$$\Phi_c = \sqrt{2} M_{Pl} \Phi$$

In Eq. (4) we use the notation

$$m \equiv r_c^{-1} = \frac{M^2}{\sqrt{2}M_{Pl}}$$

In what follows we consider sources with the energy-momentum of the form

$$T_{\mu\nu} = \rho u_\mu u_\nu$$

In the non-relativistic limit when the source velocity with respect to the rest frame of ghost condensate is small, one takes

$$T_{00} = \rho(x, t)$$

and sets all other components of $T_{\mu\nu}$ equal to zero. As a result one arrives at the following source term in the action

$$\mathcal{L}_s = -\frac{\sqrt{2}}{M_{Pl}}\Phi_c\rho \quad (5)$$

After inverting the (2×2) matrix in Eq. (4) one finds that the gravitational potential of a source can be written as a sum of two terms

$$\Phi = \Phi_N + \Delta\Phi \quad (6)$$

where Φ_N is the conventional Newtonian potential and

$$\Delta\Phi(x, t) = -\frac{2G}{\pi t_c} \int^t dt' \int d^3x' \frac{\rho(t', x')}{|x - x'|} I\left(\frac{t - t'}{t_c}, m|x - x'|\right) \quad (7)$$

The “propagator” $I(T, R)$ is

$$I(T, R) = \int_0^\infty \frac{du}{\sqrt{u^2 - 1}} \sin\left(Tu\sqrt{u^2 - 1}\right) \sin Ru \quad (8)$$

In Appendix we calculate various asymptotics of this function. The results are summarized as follows

$$I(T, R) = \frac{1}{2}\sqrt{\frac{\pi}{T}} \exp\left(\frac{T}{2} - \frac{R^2}{8T}\right) \sin \frac{R}{\sqrt{2}}, \quad R \ll T, \quad T \gg 1 \quad (9a)$$

$$I(T, R) = \sqrt{\frac{\pi T}{2}} \frac{\cos\left(\frac{R^2}{4T} + \frac{T}{2}\right) + \sin\left(\frac{R^2}{4T} + \frac{T}{2}\right)}{R}, \quad R \gg T, \quad R \gg 1 \quad (9b)$$

$$I(T, R) \propto TR, \quad R, T \ll 1 \quad (9c)$$

It is straightforward to check numerically that in the intermediate regions the function $I(R, T)$ smoothly interpolates between the different asymptotics.

3 Gravitational potentials of moving sources

3.1 Point-like mass

Now we are ready to calculate the gravitational potentials induced by different moving matter sources. To begin with, let us find the potential of a point-like mass μ moving with velocity $v \ll 1$ with respect to ghost condensate along the z axis. The corresponding mass density is equal to

$$\rho(x, t) = \mu \delta^2(y) \delta(z - vt)$$

where $y_{1,2}$ are transverse coordinates. Plugging this source into Eq. (7) and integrating out δ -functions one gets

$$\Delta \Phi_\delta(y, z, t) = -\frac{2G\mu}{\pi v t_c} \int_0^{vt} \frac{dz'}{r(z')} I\left(\frac{vt - z'}{vt_c}, \frac{r(z')}{r_c}\right) \quad (10)$$

where

$$r(z') = \sqrt{y^2 + (z' - z)^2}$$

and

$$y^2 = y_1^2 + y_2^2$$

is the distance from the observer to the trajectory of the source. The potential (10) crucially depends on whether the distance to the source trajectory y measured in units of the characteristic length scale r_c is large or small compared to the time interval (measured in units of t_c)

$$T(t, z) = \frac{vt - z}{vt_c}$$

between the moment of observation and the moment when the source was close to the point of observation³. For

$$y \gg r_c T \quad (11)$$

one makes use of the asymptotics Eq. (9b). The integral in Eq. (10) is saturated in the interval of width $\sim r_c \sqrt{T}$ near the point $z' = z$, and one arrives at the following expression for the potential in this case (barring an extremely slowly varying y -independent phase of oscillations)

$$\Delta \Phi_\delta = -\frac{2\sqrt{2}G\mu T r_c^2}{y^2 v t_c} \sin \frac{y^2}{4r_c^2 T} \quad (12)$$

³In what follows we assume that $0 < z < vt$, i.e. that the source has really passed the nearest point to the observer. It is straightforward to work out the expressions for the potential in other cases, which are less interesting.

If, on the other hand,

$$y \ll r_c T \quad (13)$$

and $T \gg 1$, then using short distance asymptotics (9a) of the propagator one obtains for the potential

$$\Delta\Phi_\delta = -\frac{G\mu}{\sqrt{\pi}vt_c} \int_0^{vt} \frac{dz'}{r(z')\sqrt{T(t, z')}} \sin \frac{r(z')}{\sqrt{2}r_c} e^{T(t, z')/2} \quad (14)$$

This integral is also saturated in the small interval near $z' = z$,

$$|z - z'| \ll r_c T$$

Then, to further simplify the integral in Eq. (14), one notes that for reasonable values of the parameters one has

$$\frac{vt_c}{r_c} = \frac{2vM_{Pl}}{\alpha M} \gg 1. \quad (15)$$

Consequently, one can substitute $T(t, z')$ by $T(t, z)$ in Eq. (14). In the resulting integral one can extend the limits of integration to $z' \in (-\infty, \infty)$ and in this way one arrives at the following expression for the potential

$$\Delta\Phi_\delta = -\sqrt{\frac{\pi}{T}} \frac{G\mu}{2vt_c} e^{T/2} J_0 \left(\frac{y}{\sqrt{2}r_c} \right) \quad (16)$$

This result is somewhat different from one obtained in Ref. [4]. Actually, it is easy to see that there is no contradiction. To obtain the latter result from our Eq. (14) consider the limit of extremely low velocity, such that

$$vt \ll y.$$

In particular, this implies that a condition opposite to our Eq. (15) holds. Then the interval of integration in Eq. (14) is small and one can set $r(z') = r(z)$. The resulting integral gives

$$\Delta\Phi_\delta \propto G \frac{\mu}{\sqrt{T}r} e^{T/2} \sin \frac{r}{\sqrt{2}r_c}$$

This result is in agreement with Ref. [4], as it should be.

To summarize our results, the gravitational potential of the moving point-like mass in the linear approximation can be described as follows. There is a narrow cone behind the source (“track”), of angle

$$\alpha = r_c/(vt_c)$$

where the potential oscillates in the transverse direction and exponentially grows backward along the axis of the cone. The period of oscillation is equal to $2\sqrt{2}\pi r_c$. The amplitude of oscillations decreases as $1/\sqrt{y}$ inside the cone, as the observer moves away from its axis. Outside the track there is a wave zone, where nearly cylindrical outgoing wave of the gravitational potential is present. As one observes by expanding the argument of sine in Eq. (12) near a given space-time point, the frequency ω and the inverse wavelength λ^{-1} of the wave grow with the distance from the trajectory of the source,

$$\omega \sim \frac{y^2}{r_c^2 T^2 t_c}, \quad \lambda^{-1} \sim \frac{y}{r_c^2 T} \quad (17)$$

The amplitude of this wave decreases as $1/y^2$. At small times $t < t_c$ the exponential track is absent, and there is the outgoing cylindrical wave only.

3.2 Source of finite size

To discuss the observational consequences of ghost condensate, one should extend the previous analysis to sources of finite size. Indeed, the potentials (12), (16) rapidly oscillate in space, so one expects that, unlike in conventional gravity, the amplitude of the potential in the track is not just proportional to the mass of the source. This amplitude is expected to be smaller for large sources due to the effect of averaging. For simplicity we consider cylindrically symmetric sources described by the following mass density,

$$\rho_c = \frac{\mu}{L^2 H} f(y/L) g((z - vt)/H), \quad (18)$$

where the functions f and g characterize the transverse and longitudinal profiles of the source, with L and H being the corresponding characteristic length scales. μ is the total mass of the source, so that the following normalization conditions are assumed,

$$\int dz g(z) = \int d^2 y f(y) = 1.$$

To calculate the potential $\Delta\Phi_c$ of the source (18) one integrates the density profile $\rho_c(x)$ of the source with the potential induced by a point-like source,

$$\Delta\Phi_c(x) = \int d^3 x' \rho_c(x') \Delta\Phi_\delta(x - x') \quad (19)$$

Let us begin with the case

$$t/t_c \gg 1, \quad \text{i.e. } T \gg 1$$

and consider the track region, where the potential is exponentially large. Making use of Eq. (16) one gets

$$\Delta\Phi_c = -\sqrt{\pi}\frac{G\mu}{2vt_cL^2H}e^{T/2}A(y)B(z) \quad (20)$$

where the functions A and B are given by the following integrals

$$A(y) = \int_{|y-y'|\lesssim r_cT} d^2y' f(y'/L) J_0\left(\frac{|y-y'|}{\sqrt{2}r_c}\right) \quad (21)$$

and

$$B(z) = \int_{z-vt}^z \frac{dz'}{\sqrt{T(t, z-z')}} g(z'/H) e^{z'/2vt_c} \quad (22)$$

In what follows we adopt a natural assumption that $L \sim H$.

Let us first consider the case of large sources,

$$L, H \gg r_cT \quad (23)$$

The condition (23) implies that *the diameter of the track of the source is equal to the size of the source itself*.

In the integral (21), one can substitute $f(y'/L)$ by $f(y/L)$. As a result one obtains in this case

$$A(y) \simeq 2\pi f(y/L) \int_0^{Tr_c} dy' y' J_0\left(\frac{y'}{\sqrt{2}r_c}\right) = 2\sqrt{2}\pi r_c^2 T f(y/L) J_1\left(\frac{T}{\sqrt{2}}\right) \quad (24)$$

To estimate the function $B(z)$ let us take the source with the step profile in z -direction,

$$g(z/H) = \begin{cases} 1, & 0 < z < H \\ 0, & z < 0 \text{ or } z > H \end{cases}$$

Then

$$B(z) \simeq \frac{b_0}{\sqrt{T(t, \tilde{z})}} e^{\tilde{z}/2vt_c}, \quad (25)$$

where

$$\tilde{z} = \min(H, z), \quad b_0 = \min(H, z, vt_c)$$

Plugging Eqs. (24), (25) into the general expression (20) one finally obtains the following result for the potential inside the track of the large homogeneous source

$$\Delta\Phi_c \simeq -(2\pi)^{3/2} \frac{G\mu r_c^2 T(t, z) b_0 f(y/L)}{L^2 H v t_c \sqrt{T(t, \tilde{z})}} \exp \frac{T(t, \tilde{z})}{2} J_1\left(\frac{T(t, \tilde{z})}{\sqrt{2}}\right) \quad (26)$$

By comparing Eq. (26) to Eq. (16) one observes that the potential inside the track of the large source is suppressed as compared to the potential inside the track of a point-like source of the same mass by a factor

$$\frac{\Delta\Phi_c}{\Delta\Phi_\delta} \simeq \frac{r_c^2 T^{1/2} b_0}{L^2 H} < \frac{r_c^2 T^{1/2}}{L^2} \quad (27)$$

The meaning of this result is easy to understand qualitatively. At a given point inside the track, only a part of the source, whose transverse size is of order $(r_c T)$ contributes to the potential. This yields a geometrical suppression factor $(r_c T/L)^2$. The suppression by an extra factor of $T^{-3/2}$ comes from the integration of the rapidly oscillating function in Eq. (24).

We see that for large homogeneous sources the transverse profile of the potential inside the track is just proportional to the transverse profile of the source, i.e. this potential is slowly changing in the transverse direction. This potential slowly oscillates with the exponentially growing amplitude along the trajectory of the source.

Let us discuss now what happens with sources of smaller size. Clearly, if the size of the source is much smaller than r_c , then the point-like approximation discussed above is applicable. So, here we consider sources of the intermediate size,

$$r_c \ll L, \quad H \ll r_c T$$

In this case it is no longer possible to pick the profile of the source $f(y'/L)$ out of the integral in Eq. (21). To estimate the suppression factor, let us take this profile in the form of the step function. Also, for simplicity, let us calculate just the value of the function $A(y)$ at the origin $y = 0$,

$$A(0) = \int_0^L d^2 y' J_0 \left(\frac{y'}{\sqrt{2} r_c} \right) = 2\sqrt{2}\pi L r_c J_1 \left(\frac{L}{\sqrt{2} r_c} \right) \quad (28)$$

For the function $B(z)$ one can still use the estimate (25). Then, proceeding as above, we obtain for the suppression factor in this case

$$\frac{\Delta\Phi_c}{\Delta\Phi_\delta} < \left(\frac{r_c}{L} \right)^{3/2} \quad (29)$$

Let us discuss now the effect of the finite size of the source in the wave zone. It is rather clear that suppression similar to that discussed above should be present in this region as well. To see this, let us plug the expression (12) for the potential of the point-like source into Eq. (19). One finds that the potential of the cylindrical source in the wave zone is given

by

$$\Delta\Phi_c = -\frac{G\mu r_c^2}{vt_c L^2 H} A(y, z), \quad (30)$$

where

$$A(y, z) = \int \frac{d^2 y' dz' f(y'/L) g(z'/H)}{|y - y'|^2} T(t, z') \sin \frac{|y - y'|^2}{4r_c^2 T(t, z')} \quad (31)$$

Now, for simplicity, we restrict our consideration to the case when the distance from the source trajectory to the observer is much larger than the size of the source, $y \gg L, H$. Then, in the non-oscillating part of the integrand in Eq. (31) one can set $|y - y'| \sim y$. In the resulting integral one performs the integration over the polar angle in the y' -plane, and obtains

$$A(y, z) = \frac{2\pi}{y^2} \int y' dy' dz' f(y'/L) g(z'/H) T(t, z - z') J_0 \left(\frac{2yy'}{4r_c^2 T(t, z - z')} \right) \sin \frac{|y - y'|^2}{4r_c^2 T(t, z - z')} \quad (32)$$

Now, to estimate the value of the integral over the radial variable y' let us take the profile in the transverse direction to be a Gaussian,

$$f(y/L) = \frac{1}{\pi} e^{-y^2/L^2}.$$

Then the integral over y' gives two different terms. One of them comes from the integration over large values of y' and is exponentially suppressed at large distances from the source, the second comes from the integration over a tiny region near the origin, $y' \lesssim \frac{r_c^2 T}{y}$. Schematically, barring coefficients of order one and constant phases of oscillations, one has

$$A(y, z) = \frac{2\pi}{y^2} \int dz' g(z'/H) \left[\left(\frac{r_c^2 T'}{y} \right)^2 \sin \frac{y^2}{4r_c^2 T'} + T'^{3/2} r_c^2 e^{-y^2/L^2} \sin \frac{4y^2 r_c^2 T'}{L^4} \right] \quad (33)$$

where T' stands for $T(t, z - z')$.

Using Eq. (33) one arrives at the following upper bound on the absolute value of the function $A(y, z)$,

$$|A(y, z)| \lesssim \frac{2\pi H}{y^2} \left[\left(\frac{r_c^2 T}{y} \right)^2 + T^{3/2} r_c^2 e^{-y^2/L^2} \right] \quad (34)$$

Plugging this upper bound into Eq. (30) and comparing the result with the potential (12) for the point source, one finds that in the wave zone

$$\frac{\Delta\Phi_c}{\Delta\Phi_\delta} < \left(\frac{r_c^2 T}{Ly} \right)^2 + T^{1/2} \frac{r_c^2}{L^2} e^{-y^2/L^2} \quad (35)$$

i.e., the suppression of the potential in the wave zone relative to the potential of the point source of the same mass is even stronger than in the track region.

4 Discussion and conclusions

Now we are at a point to discuss possible observational signatures of ghost condensate, taking into account the effect of non-zero velocity of all stellar objects in the rest frame of ghost condensate. Let us start with a few preliminary remarks. From the phenomenological point of view, ghost condensate (at the linearized level) has the characteristic time and length scales t_c and r_c . These parameters are related to the two microscopic parameters, mass scale M and dimensionless parameter α , as written in Eqs. (2), (3). The allowed deviation of the latter parameter from unity is determined by the amount of fine-tuning that one is ready to introduce in the theory. We are going to be rather generous in this respect; in fact, our discussion is quite flexible and as large values of α as 10^{10} will not affect it significantly.

An exhaustive phenomenological analysis of the theory would have resulted in the exclusion plot in the (t_c, r_c) -parameter space, and in the detailed discussion of the characteristic observational signatures for different allowed regions. We believe that such an analysis deserves a separate publication. Our purpose here is to discuss qualitative features of the ghost condensate phenomenology, stressing the crucial role of the effect of finite velocity. Our claims are the following.

1. It is very unlikely to observe ghost condensate with $t_c \sim t_U$, where t_U is the present age of the Universe. In other words, it is crucial for the observability of ghost condensate that it enters the regime in which tracks with exponentially enhanced field are present.
2. The tracks of compact massive objects (stars) become pronounced earlier than the tracks of the supermassive objects of small density (galaxies).
3. The chance to observe ghost condensate is larger for larger values of r_c .
4. Relatively promising ways of searching for tracks in ghost condensate are: (i) search for “mad” stars which feel the gravitational field of the tracks of other stars; (ii) microlensing observations and (iii) gravitational wave experiments.
5. It may happen that t_c is so small that tracks of some objects are already in the non-linear (quantum?) regime and, still, we have not noticed the presence of ghost condensate so far. Consequently, it may happen that to fully understand the phenomenology of ghost condensate one needs the details of the UV completion in the ghost sector.

Let us first explain why it is unlikely to observe ghost condensate for large characteristic time scales, $t_c \gtrsim t_U$. In this regime, tracks with exponentially enhanced field did not have

enough time to develop, so the only potentially observable effects are due to the gravitational potential in the wave zone. Let us first assume that the characteristic size r_c is somewhat larger than the size of a typical star like the Sun,

$$r_c \gtrsim 10^6 \text{ km}$$

This actually implies that the parameter α is quite large, $\alpha \gtrsim 10^5$; however, as we will see, the chance to detect ghost condensate is even lower for smaller values of r_c . Then, to estimate the “extra” gravitational potential $\Delta\Phi$ (see, Eq. (6)) of a star whose trajectory was at a distance y_0 to the current location of the Earth (in the rest frame of ghost condensate) we make use of the expression (12) for the potential of a point-like source in the wave zone,

$$\Delta\Phi \sim 10^{-20} \left(\frac{\mu}{M_\odot} \right) \left(\frac{10^{-3}}{v} \right) \left(\frac{r_c}{y_0} \right)^2 \sin \frac{y_0 \Delta y}{2r_c^2} \quad (36)$$

where we set $T = 1$, expanded the phase of oscillations in Eq. (12) near a given space-time point setting

$$y = y_0 + \Delta y$$

neglected extremely slow variation of this phase in time and in z -direction and dropped the constant shift of this phase. Equation (36) applies to the rest frame of ghost condensate, while in the rest frame of the Earth the gravitational field has the form of a wave of the amplitude $\sim 10^{-20}$ and frequency

$$\nu = \frac{v_r y_0}{4\pi r_c^2} \simeq 2 \cdot 10^{-5} \text{ Hz} \left(\frac{v_r}{10^{-3}} \right) \left(\frac{y_0}{r_c} \right) \left(\frac{10^6 \text{ km}}{r_c} \right) \quad (37)$$

where v_r is the Earth velocity in the direction transverse to the star trajectory. Note that this is a scalar wave unlike the tensor waves of the Einstein theory. The gravity waves of such a low frequency are in principle accessible to the LISA project (see, e.g., Refs. [6, 7] for reviews of the gravitational wave experiments), however, its sensitivity at these frequencies is at the level⁴ $\Delta\Phi \sim 10^{-17}$.

Furthermore, it is straightforward to see that the probability $p(r_c)$ to have even this weak signal is extremely low. Indeed, to estimate this probability, note first that the probability

⁴It is worth noting that gravitational wave experiments are sensitive not to the amplitude $\Delta\Phi$ of the gravitational wave itself, but to the product $\sqrt{n}\Delta\Phi$, where n is a number of cycles produced in a logarithmic band about a given frequency. In the case at hand $n \sim \nu y_0 / v_r = y_0^2 / (4\pi r_c^2)$. This comment is practically irrelevant for our discussion.

$\mathcal{P}(r_c)$ that the distance from the current position of the Earth to the nearest trajectory of a star is smaller than r_c , is given by

$$\mathcal{P}(r_c) \sim \frac{N_{st} N_g v t_U r_c^2}{r_U^3} \sim 10^{-15} \quad (38)$$

where $N_{st} \sim N_g \sim 10^{11}$ are, respectively, the number of stars in a typical galaxy and the number of galaxies in the Hubble volume; $r_U \sim 10^{28}$ cm, $v \sim 10^{-3}$ and $r_c \sim 10^6$ km. For long enough period of observation t_o , such that the Earth travels a distance L_E much larger than r_c , the probability $p(r_c)$ is somewhat larger,

$$p(r_c) \sim \mathcal{P}(r_c) \frac{L_E}{r_c} \sim 10^4 \cdot \mathcal{P}(r_c) \left(\frac{t_o}{1 \text{ yr}} \right) \left(\frac{v}{10^{-3}} \right) \left(\frac{10^6 \text{ km}}{r_c} \right)$$

Since the amplitude in Eq. (36) rapidly decreases at $y_0 > r_c$, this expression determines the probability of having a non-negligible signal. Clearly, this probability is quite low.

The sensitivity of gravitational wave detectors is higher in the higher frequency bands, so one might expect better signal for smaller r_c , and hence higher frequency ν . For instance, LISA will have sensitivity at the level $\Delta\Phi \sim 10^{-20}$ in the frequency range $10^{-3} \div 10^{-1}$ Hz. The waves of amplitudes and frequencies in this range would be generated for $y \sim r_c \sim 1000$ km. In this case, in order to avoid the suppression of the amplitude due to the effect of the finite size of the source (section 3.2), one should consider very compact sources like neutron stars. But the above estimate shows that we should be extremely lucky to have a trajectory of a neutron star at a distance of order 1000 km from the LISA facility.

Let us now discuss the effects from objects of larger size, e.g. galaxies. One could expect two different types of signatures on these large distance scales. The first is the gravitational wave signals like those discussed above. The second is the modification of the gravitational dynamics at large scales due to extra contributions to the Newtonian potential. However, it is straightforward to see that the effect of averaging discussed in section 3.2 kills all these signatures unless the value of the characteristic length scale r_c (and, correspondingly, of the parameter α) is extremely large. Indeed, due to the large mass of a galaxy there is an extra factor of order 10^{12} , as compared to a star, in the estimate (36) for the gravitational potential. However, Eq. (35) tells us that there is an extra suppression by at least a factor

$$\frac{r_c^2}{L_g^2} \sim 10^{-30} \left(\frac{r_c}{1000 \text{ km}} \right)^2 \left(\frac{30 \text{ kpc}}{L_g} \right)^2 \quad (39)$$

where L_g is the size of a galaxy. We see that the effects of ghost condensate are indeed very small for large objects of small density.

The above arguments show that chance to detect ghost condensate is very low if the characteristic time scale t_c is longer than, or equal to the present age of the Universe. This forces us to discuss the regime $t_c \lesssim t_U$. This regime is rather dangerous, as the gravitational and ghost fields grow exponentially inside the tracks. Still, let us make a few general remarks, postponing the detailed discussion of the ghost condensate phenomenology in this regime for future.

If the ratio t_U/t_c is large enough, the tracks of galactic halos are very pronounced (say, the gravitational potential in the track is comparable to the typical gravitational potential between two interacting galaxies). The fraction of the Universe filled by these tracks is estimated as

$$\frac{\Delta V_h}{V_U} \sim N_g \frac{r_{halo}^2 v t_u}{t_u^3} \sim 0.05 \left(\frac{N_g}{10^{11}} \right) \left(\frac{r_{halo}}{100 \text{ kpc}} \right)^2 \left(\frac{v}{10^{-3}} \right) \quad (40)$$

where we estimated the size of a halo of a typical galaxy as 100 kpc. It is unlikely that this effect would have been unnoticed. For instance, the dynamics of a sizeable number of galaxies would have been affected by these tracks.

For not so large t_U/t_c the situation is more interesting. There is a range of parameters in which tracks of galaxies are unnoticeable, but star tracks are strong. As an example, one finds from Eqs. (36) and (39) (assuming $r_c = 1000 \text{ km}$ for definiteness) that for

$$\frac{t_U}{t_c} \sim 2 \ln 10^{20} \sim 92$$

the gravitational potentials in the tracks of neutron stars are of order one, while the gravitational potentials in the tracks of typical galactic halos are still of order 10^{-18} which is more than 10 orders of magnitude smaller than the gravitational potential of a typical galaxy at a distance of order 1 Mpc. Consequently, there is an interesting range of the characteristic time scales,

$$0.01 \lesssim \frac{t_c}{t_U} < 1 \quad (41)$$

in which star tracks, but not galactic tracks, are pronounced (up to $\Delta\Phi \sim 1$).

Normally, the existence of exponentially growing field would mean that an extreme fine-tuning of the parameters is needed to have potentially observable effects without ruling out the theory completely. Clearly, to have t_c in the range (41) one needs some fine-tuning, but not that strong as one might expect.

We suggest here three potential signatures of the star tracks. First, one can search for “mad” stars, intersecting the tracks of other stars, so that their motion is strongly affected by the gravitational field of the track (probably, just for a short period of time). Second,

it seems possible to observe tracks in the microlensing experiments (see, e.g., Ref. [8] for a review of microlensing experiments) due to the variation of the visible luminosity of a background star when the line of sight crosses the track. Finally, the gravitational wave detectors may detect a signal from the track, as discussed above.

To observe mad stars one needs a galaxy whose disc is intersecting a track of the disc of another galaxy at the moment of observation. Plugging the typical disc size $r_{disc} \sim 10$ kpc instead of r_{halo} into Eq. (40) we find that this happens for approximately one galaxy of a thousand. To get a feeling of numbers let us also estimate the fraction of the volume of such a galaxy filled by the star tracks,

$$\frac{\Delta V_s}{V_{galaxy}} \sim N_s \left(\frac{r_s}{r_{disc}} \right)^2 \sim 10^{-12} , \quad (42)$$

where $r_s \sim 10^6$ km is the radius of a typical star. This estimate is not very optimistic as it implies that there is about one mad star in the galaxy at each moment of time. Note however, that one can significantly enhance the success probability by performing monitoring of stars for a long period of time. Also it would be interesting to check whether trapping of a star by the gravitational field of a track is possible. Finally, the estimate (42) assumes that the parameter r_c is smaller than r_s so that the diameter of the star track is equal to the size of the star. At larger values of r_c this fraction is enhanced by a factor of $(r_c/r_s)^2$.

Similar considerations show that chance to observe two other signatures (microlensing and gravity waves) are rather low for $r_c < r_s$.

The above estimates show that it may happen, that while we have not noticed the presence of ghost condensate yet, the gravitational and ghost fields in the tracks of some dense objects are already very strong. Note that it follows from Eq. (4) that gravitational field $\Delta\Phi \sim 1$ corresponds to ghost field $\pi_c \sim M$, so it is unclear whether non-linear classical dynamics will result in some stable non-linear classical track solution or strong quantum effects will start operating at this point. [In particular, there is a danger that the points of the field space where excitations of ghost condensate are ghosts (i.e., have wrong sign in front of kinetic term) become accessible inside the tracks.] It seems difficult to deduce what is happening in this regime without knowing the UV completed theory of ghost condensate (in other words, without constructing a complete analog of the Higgs mechanism for gravity and not just a non-linear sigma-model).

To conclude, let us mention some other open questions and further directions of research.

First, in our treatment we did not take into account the expansion of the Universe. In the interesting case (41) this assumption seems justified because at the initial, rather short

cosmological stage when the Hubble rate was higher than the rate of the development of instabilities in ghost condensate t_c^{-1} , the Hubble friction prevented the instabilities to grow. Also, the Universe was essentially homogeneous during the fast period of expansion, while the tracks discussed here emerge due to the inhomogeneities. Still, we believe that this question deserves further study. Especially interesting would be to check the possibility to have sizeable tracks of inhomogeneities which could be present before inflationary stage.

Second, our discussion was purely non-relativistic. However, one could argue that there is a possibility that the whole observed part of the Universe is moving with respect to the rest frame of ghost condensate with velocity close to the speed of light. It would be interesting to check whether this is a consistent possibility, or one of the effects of the Hubble friction is to slow down the velocity with respect to the rest frame of ghost condensate, so the non-relativistic approximation is always justified. In the latter case it would be natural to assume that the rest frame of ghost condensate coincides with the rest frame of the CMB. If true, this assumption would imply that the motion with respect to ghost condensate is entirely determined by the peculiar velocities of galaxies and galactic clusters. Then it would be interesting to reconstruct the actual map of galactic tracks at least in the local part of the Universe. Such a map would significantly decrease the uncertainties in the observational predictions of the model.

It is worth noting also, that the effect of finite velocity of an observer with respect to the rest frame of ghost condensate should also be taken into account while discussing the limits on the direct coupling between matter fields and ghost condensate, e.g., coming from the spin-dependent forces [4] mediated by ghost condensate.

Finally, ghost condensate may be considered as a particular case of the k -essence models [9, 10]. It is interesting to check whether the effects similar to those discussed here can be present and pronounced in the models of k -essence with cosmologically relevant parameters.

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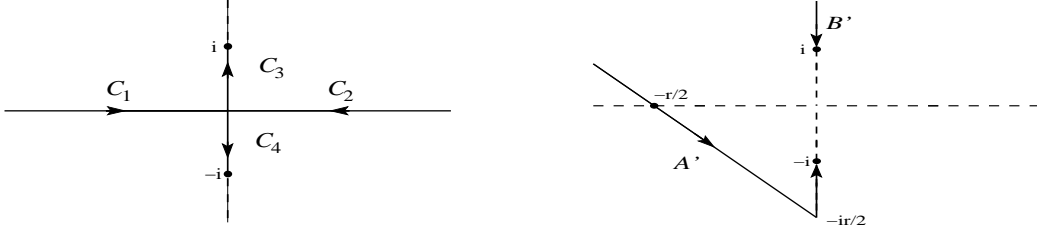


Figure 1:

Appendix A: Asymptotics of the “propagator” $I(T, R)$

The purpose of this Appendix is to find the asymptotics of the propagator $I(T, R)$ in physically interesting regions. For this purpose, let us first introduce a new integration variable

$$v = \sqrt{u^2 - 1}$$

Then it is straightforward to check that the function $I(T, R)$ takes form of the following contour integral

$$I(T, R) = \frac{1}{2} \text{Re} \int_{\mathcal{C}} \frac{dv}{\sqrt{1+v^2}} e^{i\sqrt{1+v^2}(Tv+R)}, \quad (43)$$

where the integration contour \mathcal{C} consists of four segments (see Fig. 1a)),

$$\mathcal{C} = \mathcal{C}_1 + \mathcal{C}_2 + \mathcal{C}_3 + \mathcal{C}_4$$

with

$$\mathcal{C}_1 = (-\infty, 0), \quad \mathcal{C}_2 = (\infty, 0), \quad \mathcal{C}_3 = (0, i), \quad \mathcal{C}_4 = (0, -i)$$

Now, to perform the saddle point integration we need the extrema of the exponent

$$f(v) = i\sqrt{1+v^2}(Tv+R)$$

of the integrand in Eq. (43). This function has two stationary points, given by

$$v_{\pm} = \frac{-r}{4} \pm \sqrt{r^2 - 8},$$

where we introduced the ratio

$$r \equiv \frac{R}{T}$$

At this point it is convenient to consider separately two limiting cases $r \ll 1$ and $r \gg 1$.

A1: $r \ll 1$

Let us start with the case $r \ll 1$. Here, both saddle points are near the imaginary axis,

$$v_{\pm} \approx \frac{-r}{4} \pm \frac{i}{\sqrt{2}},$$

Now, at $T \gg 1$ the function f takes large positive value at v_- . Consequently, in this range of parameters one finds that the \mathcal{C}_4 -part of the contour \mathcal{C} gives exponentially large contribution to $I(T, R)$, while other parts of the contour give at most contributions of order one. Leaving only the exponentially large piece we obtain the following saddle point approximation for the function $I(T, R)$,

$$I(T, R) = \frac{1}{2} \sqrt{\frac{\pi}{T}} \exp\left(\frac{T}{2} - \frac{R^2}{8T}\right) \sin \frac{R}{\sqrt{2}}, \quad R \ll T, \quad T \gg 1 \quad (44)$$

This exponentially large term agrees with that found in Ref. [4].

In the opposite case $T \ll 1$ function $I(T, R)$ tends to zero as $\text{const} \cdot (TR)$ as is obvious from the original expression, Eq. (8).

A2: $r \gg 1$

Let us now consider the opposite limit $r \gg 1$ (more precisely, $R/T \gg \sqrt{8}$). Here both saddle points v_{\pm} are on the real axis,

$$v_+ \approx -\frac{1}{r}, \quad v_- \approx -\frac{r}{2}.$$

First, let us note, that for $R \ll 1$ the function $I(T, R)$ again tends to zero as $\text{const} \cdot (TR)$.

Let us now turn to the regime $R \gg 1$. It is convenient to combine contours $\mathcal{C}_1, \mathcal{C}_4$ into a single contour \mathcal{A} and contours $\mathcal{C}_2, \mathcal{C}_3$ into a single contour \mathcal{B} and deform contours \mathcal{A} and \mathcal{B} to contours \mathcal{A}' and \mathcal{B}' as shown in Fig. 3. Then one can check that contributions to $I(T, R)$ that do not have exponential suppression come from the part of the contour \mathcal{A}' in the vicinity of the saddle point v_- and from the parts of the contours \mathcal{A}' and \mathcal{B}' in the vicinity of points $\mp i$. Furthermore, it is straightforward to check that the two latter contributions actually cancel out, so one is left with the saddle point contribution which is now given by

$$I = \sqrt{\frac{\pi T}{2}} \frac{\cos\left(\frac{R^2}{4T} + \frac{T}{2}\right) + \sin\left(\frac{R^2}{4T} + \frac{T}{2}\right)}{R}, \quad R \gg \sqrt{8}T, \quad R \gg 1 \quad (45)$$

To summarize, Eqs. (44), (45) provide a good approximation for the function $I(T, R)$ in all physically interesting regions.

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